

Original Article

# Optimization of Fuzzy Inventory Model using Kuhn tucker Technique by Decagonal Fuzzy Numbers

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**Abstract** - The fuzzy transportation problem is considered in this paper, where the values of transportation costs are expressed as Decagonal fuzzy numbers. It has a variety of options for transporting the goods. The aim of this research is to find the best defuzzification method for determining the lowest transportation cost.

**Keywords** – Fuzzy inventory, Kuhn tucker technique, decagonal fuzzy numbers.

## I. INTRODUCTION

Transportation theory is the name given to the study of efficient transportation and resource distribution in mathematics and economics. Gaspard Monge, a French mathematician, formalised the problem in 1781.

Unfortunately, grasping the method and deciphering the findings is a difficult job. The procedure is very complicated. The transportation issue is concerned with how to schedule development and transportation in a given sector where there are several plans at various locations and a large number of customers. The transportation issue is concerned with the movement of goods from a variety of sources, such as manufacturers, to a number of demand points, such as warehouses, which are commonly referred

to destination. Each source has a fixed capacity or availability for supplying a fixed number of units of product, and each destination has a fixed demand, which is referred to as specifications. This form of problem is commonly referred to as "The Transportation Problem" because it is frequently used to solve problems involving multiple product sources and multiple product destinations. Demand and supply for any product cannot be fixed in recent times, and they have been fluctuating due to a variety of factors. This resulted in a difference in transportation costs as well. So This research focuses on the transportation issue, with costs provided as Numbers that are vague. A fuzzy collection is a mathematical representation of ambiguous qualitative or quantitative data that is frequently produced using natural language. The model is based on a generalisation of classical set and characteristic function definitions.

A fuzzy transportation problem is considered in which the values of transportation costs are expressed as Decagonal fuzzy numbers. It has a variety of options for transporting the goods. The aim of this research is to find the best defuzzification method for determining the lowest transportation cost.

## Definition

### Fuzzy Set

Let  $X$  be a space of points with a generic element of  $X$  denoted by  $x$ . Thus  $X = \{x\}$ . A fuzzy set  $A$  in  $X$  is characterized by a membership function  $f_A(x)$  which associates with each point in  $X$  a real number in the interval  $[0,1]$  with the values of  $f_A(x)$  at  $x$  representing the "grade of membership" of  $x$  in  $A$ . Thus the nearer value of  $f_A(x)$  to unity, the higher grade of membership of  $x$  in  $A$ .



**Definition**

**Decagonal fuzzy number:**

A **Fuzzy number**  $\tilde{A}_\alpha$  is a decagonal fuzzy number denoted by  $\tilde{A}_\alpha = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10},)$  where  $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$  are real numbers and its membership function is given below,

$$\mu_{\tilde{A}_\alpha}(x) = \begin{cases} \frac{1}{4} \frac{(x-a_1)}{(a_2-a_1)}, & a_1 \leq x \leq a_2 \\ \frac{1}{4} + \frac{1}{4} \frac{(x-a_2)}{(a_3-a_2)}, & a_2 \leq x \leq a_3 \\ \frac{1}{2} + \frac{1}{4} \frac{(x-a_3)}{(a_4-a_3)}, & a_3 \leq x \leq a_4 \\ \frac{3}{4} + \frac{1}{4} \frac{(x-a_4)}{(a_5-a_4)}, & a_4 \leq x \leq a_5 \\ 1, & a_5 \leq x \leq a_6 \\ 1 - \frac{1}{4} \frac{(x-a_6)}{(a_7-a_6)}, & a_6 \leq x \leq a_7 \\ \frac{3}{4} - \frac{1}{4} \frac{(x-a_7)}{(a_8-a_7)}, & a_7 \leq x \leq a_8 \\ \frac{1}{2} - \frac{1}{4} \frac{(x-a_8)}{(a_9-a_8)}, & a_8 \leq x \leq a_9 \\ \frac{1}{4} - \frac{(x-a_9)}{(a_{10}-a_9)}, & a_9 \leq x \leq a_{10} \\ 0, & \text{otherwise} \end{cases}$$

**Ranking of Decagonal fuzzy number:**

**Fuzzy numbers directly get into the real line by using ranking method.** Let  $\tilde{A}_\alpha$  be generalized decagonal fuzzy number. The ranking of  $\tilde{A}_\alpha$  is denoted by  $R(\tilde{A}_\alpha)$  and it is calculated as follows:

$$R(\tilde{A}_\alpha) = \left[ \frac{a_1 + 3a_2 + 5a_3 + 7a_4 + 9a_5 + 9a_6 + 7a_7 + 5a_8 + 3a_9 + a_{10} +}{50} \right]$$

**The fuzzy arithmetical operations under function principle**

Suppose  $\tilde{A} = (a_1, a_2, a_3, a_4)$  and  $\tilde{B} = (b_1, b_2, b_3, b_4)$  are two trapezoidal fuzzy numbers. Then,

1. The addition of  $\tilde{A}$  and  $\tilde{B}$  is  $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$  Where  $(a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4)$  are any real numbers.
2. The multiplication of  $\tilde{A}$  and  $\tilde{B}$  is  $\tilde{A} \otimes \tilde{B} = (c_1, c_2, c_3, c_4)$ , Where  $(a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4)$  are all non zero positive real numbers, then

$$\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$$

3.  $-\tilde{B} = (-b_1, -b_2, -b_3, -b_4)$ , then the subtraction of  $\tilde{A}$  and  $\tilde{B}$  is

$$\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1) \text{ Where } (a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4) \text{ are any real numbers.}$$

4.  $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = (\frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1})$  where are all positive real

numbers. If  $(a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4)$  are all nonzero positive real numbers, then the

division of  $\tilde{A}$  and  $\tilde{B}$  is  $\tilde{A} \oslash \tilde{B} = (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1})$

5. Let  $\alpha \in \mathbb{R} \dots$  Then,

$$\alpha \geq 0, \alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4),$$

$$\alpha < 0, \alpha \otimes \tilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1).$$

**Notations:**

$A_m$  - Holding Cost

$B_n$  - Length of the plan

$C_s$  - Ordering cost

$D_t$  - Demand with time period

$q^*$  - Order quantity

$T_C$  - Total Cost

$Q^*$  - Optimal order quantity

$\tilde{T}_C$  - Fuzzy tota cost

**Crisp Sense:**

$$T_C = \frac{A_m B_n q}{2} + \frac{C_s D_t}{q}$$

Differentiate partially with respect to ‘q’

$$\Rightarrow \frac{A_m B_n}{2} = \frac{C_s D_t}{q^2}$$

$$q^2 = \frac{2C_s D_t}{A_m B_n}$$

$$q = \sqrt{\frac{2C_s D_t}{A_m B_n}}$$

**Fuzzy sense:**

$$\tilde{T}_C = \frac{\tilde{A}_m \tilde{B}_n q}{2} + \frac{\tilde{C}_s \tilde{D}_t}{q}$$

( $\tilde{A}_m, \tilde{B}_n, \tilde{C}_s, \tilde{D}_t$ ) be the decagonal fuzzy numbers.

$$\tilde{T}_C = \left[ \begin{array}{cccc} \frac{A_{m_1} B_{n_1} q}{2} + \frac{C_{s_1} D_{t_1}}{q}, & \frac{A_{m_2} B_{n_2} q}{2} + \frac{C_{s_2} D_{t_2}}{q}, & \frac{A_{m_3} B_{n_3} q}{2} + \frac{C_{s_3} D_{t_3}}{q}, & \frac{A_{m_4} B_{n_4} q}{2} + \frac{C_{s_4} D_{t_4}}{q}, \\ \frac{A_{m_5} B_{n_5} q}{2} + \frac{C_{s_5} D_{t_5}}{q}, & \frac{A_{m_6} B_{n_6} q}{2} + \frac{C_{s_6} D_{t_6}}{q}, & \frac{A_{m_7} B_{n_7} q}{2} + \frac{C_{s_7} D_{t_7}}{q}, & \frac{A_{m_8} B_{n_8} q}{2} + \frac{C_{s_8} D_{t_8}}{q}, \\ \frac{A_{m_9} B_{n_9} q}{2} + \frac{C_{s_9} D_{t_9}}{q}, & \frac{A_{m_{10}} B_{n_{10}} q}{2} + \frac{C_{s_{10}} D_{t_{10}}}{q} & & \end{array} \right]$$

By Graded mean integration representation method,

$$\frac{\partial T_C}{\partial q} = 0 \Rightarrow \frac{1}{50} \left[ \begin{array}{l} 1 \left( \frac{A_{m_1} B_{n_1} q}{2} + \frac{C_{s_1} D_{t_1}}{q} \right) + 3 \left( \frac{A_{m_2} B_{n_2} q}{2} + \frac{C_{s_2} D_{t_2}}{q} \right) + 5 \left( \frac{A_{m_3} B_{n_3} q}{2} + \frac{C_{s_3} D_{t_3}}{q} \right) + 7 \left( \frac{A_{m_4} B_{n_4} q}{2} + \frac{C_{s_4} D_{t_4}}{q} \right) + \\ 9 \left( \frac{A_{m_5} B_{n_5} q}{2} + \frac{C_{s_5} D_{t_5}}{q} \right) + 9 \left( \frac{A_{m_6} B_{n_6} q}{2} + \frac{C_{s_6} D_{t_6}}{q} \right) + 7 \left( \frac{A_{m_7} B_{n_7} q}{2} + \frac{C_{s_7} D_{t_7}}{q} \right) + 5 \left( \frac{A_{m_8} B_{n_8} q}{2} + \frac{C_{s_8} D_{t_8}}{q} \right) + \\ 3 \left( \frac{A_{m_9} B_{n_9} q}{2} + \frac{C_{s_9} D_{t_9}}{q} \right) + 1 \left( \frac{A_{m_{10}} B_{n_{10}} q}{2} + \frac{C_{s_{10}} D_{t_{10}}}{q} \right) \end{array} \right]$$

Differentiate partially with respect to q and equate them to zero,

$$\frac{\partial T_C}{\partial q} = 0 \Rightarrow \frac{1}{50} \left[ \begin{array}{l} 1 \left( \frac{A_{m_1} B_{n_1} q}{2} - \frac{C_{s_1} D_{t_1}}{q^2} \right) + 3 \left( \frac{A_{m_2} B_{n_2} q}{2} - \frac{C_{s_2} D_{t_2}}{q^2} \right) + 5 \left( \frac{A_{m_3} B_{n_3} q}{2} - \frac{C_{s_3} D_{t_3}}{q^2} \right) + 7 \left( \frac{A_{m_4} B_{n_4} q}{2} - \frac{C_{s_4} D_{t_4}}{q^2} \right) + \\ 9 \left( \frac{A_{m_5} B_{n_5} q}{2} - \frac{C_{s_5} D_{t_5}}{q^2} \right) + 9 \left( \frac{A_{m_6} B_{n_6} q}{2} - \frac{C_{s_6} D_{t_6}}{q^2} \right) + 7 \left( \frac{A_{m_7} B_{n_7} q}{2} - \frac{C_{s_7} D_{t_7}}{q^2} \right) + 5 \left( \frac{A_{m_8} B_{n_8} q}{2} - \frac{C_{s_8} D_{t_8}}{q^2} \right) + \\ 3 \left( \frac{A_{m_9} B_{n_9} q}{2} - \frac{C_{s_9} D_{t_9}}{q^2} \right) + 1 \left( \frac{A_{m_{10}} B_{n_{10}} q}{2} - \frac{C_{s_{10}} D_{t_{10}}}{q^2} \right) \end{array} \right]$$

$$q^* = \sqrt{\frac{2 \left[ 1C_{s_1} D_{t_1} + 3C_{s_2} D_{t_2} + 5C_{s_3} D_{t_3} + 7C_{s_4} D_{t_4} + 9C_{s_5} D_{t_5} + 9C_{s_6} D_{t_6} + 7C_{s_7} D_{t_7} + 5C_{s_8} D_{t_8} + 3C_{s_9} D_{t_9} + 1C_{s_{10}} D_{t_{10}} \right]}{\left[ 1A_{m_1} B_{n_1} + 3A_{m_2} B_{n_2} + 5A_{m_3} B_{n_3} + 7A_{m_4} B_{n_4} + 9A_{m_5} B_{n_5} + 9A_{m_6} B_{n_6} + 7A_{m_7} B_{n_7} + 5A_{m_8} B_{n_8} + 3A_{m_9} B_{n_9} + 1A_{m_{10}} B_{n_{10}} \right]}}$$

Kuhn-Tucker Conditions:

- (i)  $\lambda \leq 0$
- (ii)  $\nabla f p(T_c) - \lambda \nabla g(Q) = 0$
- (iii)  $\lambda_i g_i(Q) = 0, i = 1, 2, \dots, m$
- (iv)  $g_i(Q) \geq 0, i = 1, 2, \dots, m$

$$\tilde{T}_C = \frac{\tilde{A}_m \tilde{B}_n q}{2} + \frac{\tilde{C}_s \tilde{D}_t}{q}$$

$$q = (q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10},)$$

$$P(T_C) = \frac{1}{50} \left[ \begin{aligned} &1 \left( \frac{A_{m_1} B_{n_1} q}{2} + \frac{C_{s_1} D_{t_1}}{q_{10}} \right) + 3 \left( \frac{A_{m_2} B_{n_2} q}{2} + \frac{C_{s_2} D_{t_2}}{q_9} \right) + 5 \left( \frac{A_{m_3} B_{n_3} q}{2} + \frac{C_{s_3} D_{t_3}}{q_8} \right) + 7 \left( \frac{A_{m_4} B_{n_4} q}{2} + \frac{C_{s_4} D_{t_4}}{q_7} \right) + \\ &9 \left( \frac{A_{m_5} B_{n_5} q}{2} + \frac{C_{s_5} D_{t_5}}{q_6} \right) + 9 \left( \frac{A_{m_6} B_{n_6} q}{2} + \frac{C_{s_6} D_{t_6}}{q_5} \right) + 7 \left( \frac{A_{m_7} B_{n_7} q}{2} + \frac{C_{s_7} D_{t_7}}{q_4} \right) + 5 \left( \frac{A_{m_8} B_{n_8} q}{2} + \frac{C_{s_8} D_{t_8}}{q_3} \right) + \\ &3 \left( \frac{A_{m_9} B_{n_9} q}{2} + \frac{C_{s_9} D_{t_9}}{q_2} \right) + 1 \left( \frac{A_{m_{10}} B_{n_{10}} q}{2} + \frac{C_{s_{10}} D_{t_{10}}}{q_1} \right) \end{aligned} \right]$$

$$0 \leq q_1 \leq q_2 \leq q_3 \leq q_4 \leq q_5 \leq q_6 \leq q_7 \leq q_8 \leq q_9 \leq q_{10}$$

It can be written as,

$$\begin{aligned} q_2 - q_1 &\geq 0, q_3 - q_2 \geq 0, q_4 - q_3 \geq 0, q_5 - q_4 \geq 0, \\ q_6 - q_5 &\geq 0, q_7 - q_6 \geq 0, q_8 - q_7 \geq 0, q_9 - q_8 \geq 0, \\ q_{10} - q_9 &\geq 0, q_1 \geq 0, \end{aligned}$$

,

**Condition 1:**

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10} \leq 0$$

**Condition 2:**

$$\begin{aligned} &\frac{\partial}{\partial q_1} (P(T_c)) - \lambda_1 \frac{\partial}{\partial q_1} (g_1(Q)) - \lambda_2 \frac{\partial}{\partial q_2} (g_2(Q)) - \lambda_3 \frac{\partial}{\partial q_3} (g_3(Q)) - \lambda_4 \frac{\partial}{\partial q_4} (g_4(Q)) \\ &- \lambda_5 \frac{\partial}{\partial q_5} (g_5(Q)) - \lambda_6 \frac{\partial}{\partial q_6} (g_6(Q)) - \lambda_7 \frac{\partial}{\partial q_7} (g_7(Q)) - \lambda_8 \frac{\partial}{\partial q_8} (g_8(Q)) \\ &- \lambda_9 \frac{\partial}{\partial q_9} (g_9(Q)) - \lambda_{10} \frac{\partial}{\partial q_{10}} (g_{10}(Q)) \end{aligned}$$

Differentiate  $q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}$  and equate them to zero,

$$\begin{aligned} & \frac{1}{6} \left[ \frac{A_{m_1} B_{n_1}}{2} - \frac{C_{s_{10}} D_{t_{10}}}{q_1^2} \right] - \lambda_1 \frac{\partial}{\partial q_1} (q_2 - q_1) - \lambda_2 \frac{\partial}{\partial q_2} (q_3 - q_2) - \lambda_3 \frac{\partial}{\partial q_1} (q_4 - q_3) - \\ & \lambda_4 \frac{\partial}{\partial q_1} (q_5 - q_4) - \lambda_5 \frac{\partial}{\partial q_1} (q_6 - q_5) - \lambda_6 \frac{\partial}{\partial q_1} (q_7 - q_6) - \lambda_7 \frac{\partial}{\partial q_1} (q_8 - q_7) - \lambda_8 \frac{\partial}{\partial q_1} (q_9 - q_8) \\ & - \lambda_9 \frac{\partial}{\partial q_1} (q_{10} - q_9) - \lambda_{10} \frac{\partial}{\partial q_1} q_1 + \lambda_1 - \lambda_{10} = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{6} \left[ \frac{3A_{m_2} B_{n_2}}{2} - \frac{3C_{s_9} D_{t_9}}{q_2^2} \right] - \lambda_1 \frac{\partial}{\partial q_2} (q_2 - q_1) - \lambda_2 \frac{\partial}{\partial q_2} (q_3 - q_2) - \lambda_3 \frac{\partial}{\partial q_2} (q_4 - q_3) \\ & - \lambda_4 \frac{\partial}{\partial q_2} (q_5 - q_4) - \lambda_5 \frac{\partial}{\partial q_2} (q_6 - q_5) - \lambda_6 \frac{\partial}{\partial q_2} (q_7 - q_6) - \lambda_7 \frac{\partial}{\partial q_2} (q_8 - q_7) - \lambda_8 \frac{\partial}{\partial q_2} (q_9 - q_8) \\ & - \lambda_9 \frac{\partial}{\partial q_2} (q_{10} - q_9) - \lambda_{10} \frac{\partial}{\partial q_2} q_1 - \lambda_1 + \lambda_2 = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{6} \left[ \frac{5A_{m_3} B_{n_3}}{2} - \frac{5C_{s_8} D_{t_8}}{q_3^2} \right] - \lambda_1 \frac{\partial}{\partial q_3} (q_2 - q_1) - \lambda_2 \frac{\partial}{\partial q_3} (q_3 - q_2) - \lambda_3 \frac{\partial}{\partial q_3} (q_4 - q_3) \\ & - \lambda_4 \frac{\partial}{\partial q_3} (q_5 - q_4) - \lambda_5 \frac{\partial}{\partial q_3} (q_6 - q_5) - \lambda_6 \frac{\partial}{\partial q_3} (q_7 - q_6) - \lambda_7 \frac{\partial}{\partial q_3} (q_8 - q_7) - \lambda_8 \frac{\partial}{\partial q_3} (q_9 - q_8) \\ & - \lambda_9 \frac{\partial}{\partial q_3} (q_{10} - q_9) - \lambda_{10} \frac{\partial}{\partial q_3} q_1 - \lambda_2 + \lambda_3 = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{6} \left[ \frac{7A_{m_4} B_{n_4}}{2} - \frac{7C_{s_7} D_{t_7}}{q_4^2} \right] - \lambda_1 \frac{\partial}{\partial q_4} (q_2 - q_1) - \lambda_2 \frac{\partial}{\partial q_4} (q_3 - q_2) - \lambda_3 \frac{\partial}{\partial q_4} (q_4 - q_3) \\ & - \lambda_4 \frac{\partial}{\partial q_4} (q_5 - q_4) - \lambda_5 \frac{\partial}{\partial q_4} (q_6 - q_5) - \lambda_6 \frac{\partial}{\partial q_4} (q_7 - q_6) - \lambda_7 \frac{\partial}{\partial q_4} (q_8 - q_7) - \lambda_8 \frac{\partial}{\partial q_4} (q_9 - q_8) \\ & - \lambda_9 \frac{\partial}{\partial q_4} (q_{10} - q_9) - \lambda_{10} \frac{\partial}{\partial q_4} q_1 + \lambda_4 - \lambda_3 = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{6} \left[ \frac{9A_{m_5} B_{n_5}}{2} - \frac{9C_{s_6} D_{t_6}}{q_5^2} \right] - \lambda_1 \frac{\partial}{\partial q_5} (q_2 - q_1) - \lambda_2 \frac{\partial}{\partial q_5} (q_3 - q_2) - \lambda_3 \frac{\partial}{\partial q_5} (q_4 - q_3) \\ & - \lambda_4 \frac{\partial}{\partial q_5} (q_5 - q_4) - \lambda_5 \frac{\partial}{\partial q_5} (q_6 - q_5) - \lambda_6 \frac{\partial}{\partial q_5} (q_7 - q_6) - \lambda_7 \frac{\partial}{\partial q_5} (q_8 - q_7) - \lambda_8 \frac{\partial}{\partial q_5} (q_9 - q_8) \\ & - \lambda_9 \frac{\partial}{\partial q_5} (q_{10} - q_9) - \lambda_{10} \frac{\partial}{\partial q_5} q_1 - \lambda_4 + \lambda_5 = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{6} \left[ \frac{9A_{m_6} B_{n_6}}{2} - \frac{9C_{s_5} D_{t_5}}{q_6^2} \right] - \lambda_1 \frac{\partial}{\partial q_6} (q_2 - q_1) - \lambda_2 \frac{\partial}{\partial q_6} (q_3 - q_2) - \lambda_3 \frac{\partial}{\partial q_6} (q_4 - q_3) \\ & - \lambda_4 \frac{\partial}{\partial q_6} (q_5 - q_4) - \lambda_5 \frac{\partial}{\partial q_6} (q_6 - q_5) - \lambda_6 \frac{\partial}{\partial q_6} (q_7 - q_6) - \lambda_7 \frac{\partial}{\partial q_6} (q_8 - q_7) - \lambda_8 \frac{\partial}{\partial q_6} (q_9 - q_8) \\ & - \lambda_9 \frac{\partial}{\partial q_6} (q_{10} - q_9) - \lambda_{10} \frac{\partial}{\partial q_6} q_1 - \lambda_5 + \lambda_6 = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{6} \left[ \frac{7A_{m_7} B_{n_7}}{2} - \frac{7C_{s_4} D_{t_4}}{q_7^2} \right] - \lambda_1 \frac{\partial}{\partial q_7} (q_2 - q_1) - \lambda_2 \frac{\partial}{\partial q_7} (q_3 - q_2) - \lambda_3 \frac{\partial}{\partial q_7} (q_4 - q_3) \\ & - \lambda_4 \frac{\partial}{\partial q_7} (q_5 - q_4) - \lambda_5 \frac{\partial}{\partial q_7} (q_6 - q_5) - \lambda_6 \frac{\partial}{\partial q_7} (q_7 - q_6) - \lambda_7 \frac{\partial}{\partial q_7} (q_8 - q_7) - \lambda_8 \frac{\partial}{\partial q_7} (q_9 - q_8) \\ & - \lambda_9 \frac{\partial}{\partial q_7} (q_{10} - q_9) - \lambda_{10} \frac{\partial}{\partial q_7} q_1 - \lambda_6 + \lambda_7 = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{6} \left[ \frac{5A_{m_8} B_{n_8}}{2} - \frac{5C_{s_3} D_{t_3}}{q_8^2} \right] - \lambda_1 \frac{\partial}{\partial q_8} (q_2 - q_1) - \lambda_2 \frac{\partial}{\partial q_8} (q_3 - q_2) - \lambda_3 \frac{\partial}{\partial q_8} (q_4 - q_3) \\ & - \lambda_4 \frac{\partial}{\partial q_8} (q_5 - q_4) - \lambda_5 \frac{\partial}{\partial q_8} (q_6 - q_5) - \lambda_6 \frac{\partial}{\partial q_8} (q_7 - q_6) - \lambda_7 \frac{\partial}{\partial q_8} (q_8 - q_7) - \lambda_8 \frac{\partial}{\partial q_8} (q_9 - q_8) \\ & - \lambda_9 \frac{\partial}{\partial q_8} (q_{10} - q_9) - \lambda_{10} \frac{\partial}{\partial q_8} q_1 - \lambda_7 + \lambda_8 = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{6} \left[ \frac{3A_{m_9} B_{n_9}}{2} - \frac{3C_{s_2} D_{t_2}}{q_9^2} \right] - \lambda_1 \frac{\partial}{\partial q_9} (q_2 - q_1) - \lambda_2 \frac{\partial}{\partial q_9} (q_3 - q_2) - \lambda_3 \frac{\partial}{\partial q_9} (q_4 - q_3) \\ & - \lambda_4 \frac{\partial}{\partial q_9} (q_5 - q_4) - \lambda_5 \frac{\partial}{\partial q_9} (q_6 - q_5) - \lambda_6 \frac{\partial}{\partial q_9} (q_7 - q_6) - \lambda_7 \frac{\partial}{\partial q_9} (q_8 - q_7) - \lambda_8 \frac{\partial}{\partial q_9} (q_9 - q_8) \\ & - \lambda_9 \frac{\partial}{\partial q_9} (q_{10} - q_9) - \lambda_{10} \frac{\partial}{\partial q_9} q_1 - \lambda_8 + \lambda_9 = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{6} \left[ \frac{A_{m_{10}} B_{n_{10}}}{2} - \frac{C_{s_1} D_{t_1}}{q_{10}^2} \right] - \lambda_1 \frac{\partial}{\partial q_{10}} (q_2 - q_1) - \lambda_2 \frac{\partial}{\partial q_{10}} (q_3 - q_2) - \lambda_3 \frac{\partial}{\partial q_{10}} (q_4 - q_3) \\ & - \lambda_4 \frac{\partial}{\partial q_{10}} (q_5 - q_4) - \lambda_5 \frac{\partial}{\partial q_{10}} (q_6 - q_5) - \lambda_6 \frac{\partial}{\partial q_{10}} (q_7 - q_6) - \lambda_7 \frac{\partial}{\partial q_{10}} (q_8 - q_7) - \lambda_8 \frac{\partial}{\partial q_{10}} (q_9 - q_8) \\ & - \lambda_9 \frac{\partial}{\partial q_{10}} (q_{10} - q_9) - \lambda_{10} \frac{\partial}{\partial q_{10}} q_1 - \lambda_9 + \lambda_{10} = 0 \end{aligned}$$

**Condition 3:**

The condition is  $[\lambda_{10} g_{10}(Q)] = 0 \Rightarrow \lambda_{10} q_1 = 0$

**Condition 4:**

$$q_2 - q_1 \geq 0, q_3 - q_2 \geq 0, q_4 - q_3 \geq 0, q_5 - q_4 \geq 0,$$

$$q_6 - q_5 \geq 0, q_7 - q_6 \geq 0, q_8 - q_7 \geq 0, q_9 - q_8 \geq 0,$$

$$q_{10} - q_9 \geq 0, q_1 \geq 0,$$

,

Here we know that  $q_1 \geq 0, \lambda_{10}q_1 = 0$ , so we get  $\lambda_{10} = 0$ ,

We can replace  $q_2$  by  $q_1, q_3$  by  $q_2, q_4$  by  $q_3, q_5$  by  $q_4, q_6$  by  $q_5, q_7$  by  $q_6, q_8$  by  $q_7, q_9$  by  $q_8, q_{10}$  by  $q_9$  then  $[q_1 = q_2 = q_3 = q_4 = q_5 = q_6 = q_7 = q_8 = q_9 = q_{10}]$

By adding condition 3 we get,

$$Q^* = \sqrt{\frac{2[1C_{s_1}D_{t_1} + 3C_{s_2}D_{t_2} + 5C_{s_3}D_{t_3} + 7C_{s_4}D_{t_4} + 9C_{s_5}D_{t_5} + 9C_{s_6}D_{t_6} + 7C_{s_7}D_{t_7} + 5C_{s_8}D_{t_8} + 3C_{s_9}D_{t_9} + 1C_{s_{10}}D_{t_{10}}]}{[1A_{m_1}B_{n_1} + 3A_{m_2}B_{n_2} + 5A_{m_3}B_{n_3} + 7A_{m_4}B_{n_4} + 9A_{m_5}B_{n_5} + 9A_{m_6}B_{n_6} + 7A_{m_7}B_{n_7} + 5A_{m_8}B_{n_8} + 3A_{m_9}B_{n_9} + 1A_{m_{10}}B_{n_{10}}]}}$$

**Numerical Example:**

**Crisp sense:**

$$C_s = 5000$$

$$D_t = 3000$$

$$A_m = 500$$

$$B_n = 700$$

**Case I:**

$$q = \sqrt{\frac{2C_s D_t}{A_m B_n}}$$

$$q^* = 9.257$$

**Case II:**

$$T_C = \frac{A_m B_n q}{2} + \frac{C_s D_t}{q}$$

$$T_C = Rs.3240370.38$$

**Fuzzy sense:**

$$C_{s_1} = 500, C_{s_2} = 1500, C_{s_3} = 2500, C_{s_4} = 3500, C_{s_5} = 4500,$$

$$C_{s_6} = 5500, C_{s_7} = 6500, C_{s_8} = 7500, C_{s_9} = 8500, C_{s_{10}} = 9500.$$



$$D_{t_1} = 750, D_{t_2} = 1250, D_{t_3} = 1750, D_{t_4} = 2250, D_{t_5} = 2750, \\ D_{t_6} = 3250, D_{t_7} = 3750, D_{t_8} = 4250, D_{t_9} = 4750, D_{t_{10}} = 5250.$$

$$A_{m_1} = 50, A_{m_2} = 150, A_{m_3} = 250, A_{m_4} = 350, A_{m_5} = 450, \\ A_{m_6} = 550, A_{m_7} = 650, A_{m_8} = 750, A_{m_9} = 850, A_{m_{10}} = 950.$$

$$B_{n_1} = 430, B_{n_2} = 490, B_{n_3} = 550, B_{n_4} = 610, B_{n_5} = 670, \\ B_{n_6} = 730, B_{n_7} = 790, B_{n_8} = 850, B_{n_9} = 910, B_{n_{10}} = 970.$$

**Case I:**

$$Q^* = \sqrt{\frac{2[1C_{s_1}D_{t_1} + 3C_{s_2}D_{t_2} + 5C_{s_3}D_{t_3} + 7C_{s_4}D_{t_4} + 9C_{s_5}D_{t_5} + 9C_{s_6}D_{t_6} + 7C_{s_7}D_{t_7} + 5C_{s_8}D_{t_8} + 3C_{s_9}D_{t_9} + 1C_{s_{10}}D_{t_{10}}]}{[1A_{m_1}B_{n_1} + 3A_{m_2}B_{n_2} + 5A_{m_3}B_{n_3} + 7A_{m_4}B_{n_4} + 9A_{m_5}B_{n_5} + 9A_{m_6}B_{n_6} + 7A_{m_7}B_{n_7} + 5A_{m_8}B_{n_8} + 3A_{m_9}B_{n_9} + 1A_{m_{10}}B_{n_{10}}]}}$$

$$Q^* = 6.76$$

**Case II:**

$$\frac{\partial T_C}{\partial q} = \frac{1}{50} \left[ \begin{aligned} &1 \left( \frac{A_{m_1}B_{n_1}q}{2} + \frac{C_{s_1}D_{t_1}}{q} \right) + 3 \left( \frac{A_{m_2}B_{n_2}q}{2} + \frac{C_{s_2}D_{t_2}}{q} \right) + 5 \left( \frac{A_{m_3}B_{n_3}q}{2} + \frac{C_{s_3}D_{t_3}}{q} \right) + 7 \left( \frac{A_{m_4}B_{n_4}q}{2} + \frac{C_{s_4}D_{t_4}}{q} \right) + \\ &9 \left( \frac{A_{m_5}B_{n_5}q}{2} + \frac{C_{s_5}D_{t_5}}{q} \right) + 9 \left( \frac{A_{m_6}B_{n_6}q}{2} + \frac{C_{s_6}D_{t_6}}{q} \right) + 7 \left( \frac{A_{m_7}B_{n_7}q}{2} + \frac{C_{s_7}D_{t_7}}{q} \right) + 5 \left( \frac{A_{m_8}B_{n_8}q}{2} + \frac{C_{s_8}D_{t_8}}{q} \right) + \\ &3 \left( \frac{A_{m_9}B_{n_9}q}{2} + \frac{C_{s_9}D_{t_9}}{q} \right) + 1 \left( \frac{A_{m_{10}}B_{n_{10}}q}{2} + \frac{C_{s_{10}}D_{t_{10}}}{q} \right) \end{aligned} \right]$$

$$T_C^* = 3239258.21$$

**CONCLUSION**

This paper describes a reflection for the elemental base of fuzzy sets, as well as its operations, as well as the decagonal fuzzy numbers used for fuzzifying inventory parameters and decision variables through discrepancy between crisp and fuzzy instances. For optimising the EOQ inventory model and minimising the total expected cost function, the Kuhn-Tucker method was used. A rating function for relating to approach in fuzzy sets was used to further investigate the model.

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